Questions

Q1.

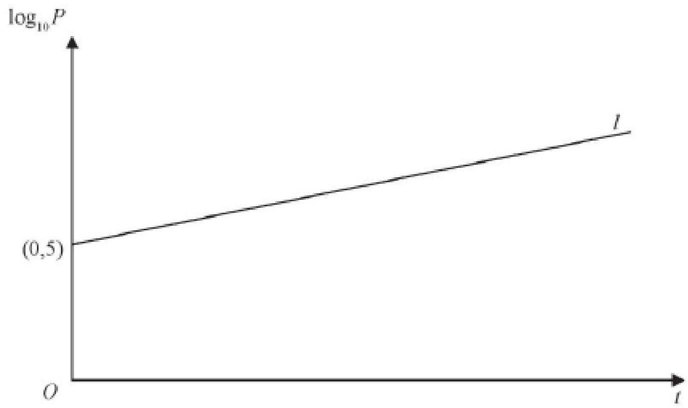


Figure 2

A town's population, P, is modelled by the equation $P = ab^t$, where a and b are constants and t is the number of years since the population was first recorded.

The line l shown in Figure 2 illustrates the linear relationship between t and $\log_{10}P$ for the population over a period of 100 years.

The line l meets the vertical axis at (0, 5) as shown. The gradient of l is $\frac{1}{200}$.

(a) Write down an equation for l.

(2)

(b) Find the value of a and the value of b.

(4)

(c) With reference to the model interpret

(i) the value of the constant a,
(ii) the value of the constant b

(2)

(d) Find

(i) the population predicted by the model when t = 100, giving your answer to the nearest hundred thousand,
(ii) the number of years it takes the population to reach 200 000, according to the model.

(3)

(e) State two reasons why this may not be a realistic population model.

(2)

(Total for question = 13 marks)

Q2.

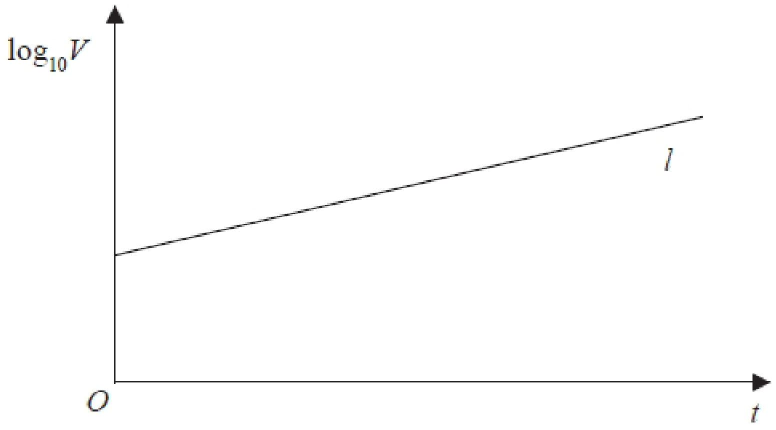


Figure 3

The value of a rare painting, £V, is modelled by the equation $V = pq^t$, where p and q are constants and t is the number of years since the value of the painting was first recorded on 1st January 1980.

The line l shown in Figure 3 illustrates the linear relationship between t and $\log_{10}V$ since 1st January 1980.

The equation of line l is $\log_{10}V = 0.05t + 4.8$

(a) Find, to 4 significant figures, the value of p and the value of q.

(4)

(b) With reference to the model interpret

(i) the value of the constant p,
(ii) the value of the constant q.

(2)

(c) Find the value of the painting, as predicted by the model, on 1st January 2010, giving your answer to the nearest hundred thousand pounds.

(2)

(Total for question = 8 marks)

Q3.

The value of a car, £*V*, can be modelled by the equation

$$V = 15\,700\mathrm{e}^{-0.25t} + 2300 \qquad t \in \mathbb{R},\ t \geqslant 0$$

where the age of the car is *t* years.

Using the model,

(a) find the initial value of the car.

(1)

Given the model predicts that the value of the car is decreasing at a rate of £500 per year at the instant when $t = T$,

(b) (i) show that

$$3925\mathrm{e}^{-0.25T} = 500$$

(ii) Hence find the age of the car at this instant, giving your answer in years and months to the nearest month.
(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(6)

The model predicts that the value of the car approaches, but does not fall below, £*A*.

(c) State the value of *A*.

(1)

(d) State a limitation of this model.

(1)

(Total for question = 9 marks)

Q4.

The temperature, θ°C, of a cup of tea t minutes after it was placed on a table in a room, is modelled by the equation

$$\theta = 18 + 65e^{-\frac{t}{8}} \qquad t \geqslant 0$$

Find, according to the model,

(a) the temperature of the cup of tea when it was placed on the table,

(1)

(b) the value of t, to one decimal place, when the temperature of the cup of tea was 35°C.

(3)

(c) Explain why, according to this model, the temperature of the cup of tea could not fall to 15°C.

(1)

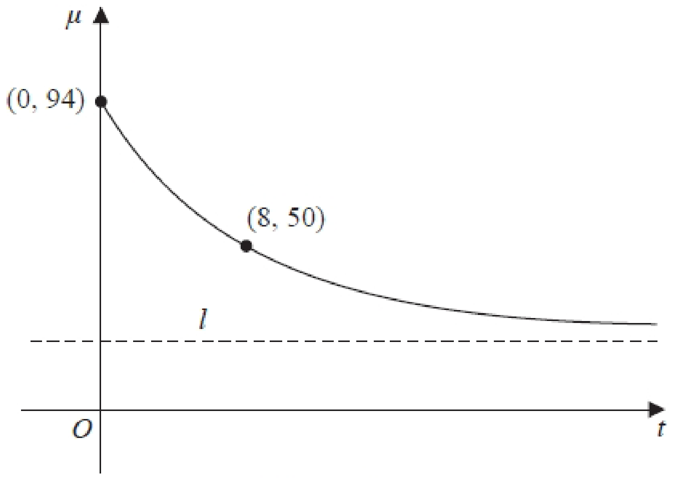


Figure 2

The temperature, μ°C, of a second cup of tea t minutes after it was placed on a table in a different room, is modelled by the equation

$$\mu = A + Be^{-\frac{t}{8}} \qquad t \geqslant 0$$

where A and B are constants.

Figure 2 shows a sketch of μ against t with two data points that lie on the curve.

The line l, also shown on Figure 2, is the asymptote to the curve.

Using the equation of this model and the information given in Figure 2

(d) find an equation for the asymptote l.

(4)

(Total for question = 9 marks)

Q5.

An advertising agency is monitoring the number of views of an online advert.

The equation

$$\log_{10} V = 0.072t + 2.379 \qquad 1 \leqslant t \leqslant 30, t \in \mathbb{N}$$

is used to model the total number of views of the advert, V, in the first t days after the advert went live.

(a) Show that $V = ab^t$ where a and b are constants to be found.

Give the value of a to the nearest whole number and give the value of b to 3 significant figures.

(4)

(b) Interpret, with reference to the model, the value of ab.

(1)

Using this model, calculate

(c) The total number of views of the advert in the first 20 days after the advert went live.

Give your answer to 2 significant figures.

(2)

(Total for question = 7 marks)

Q6.

The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

(Total for question = 6 marks)

Q7.

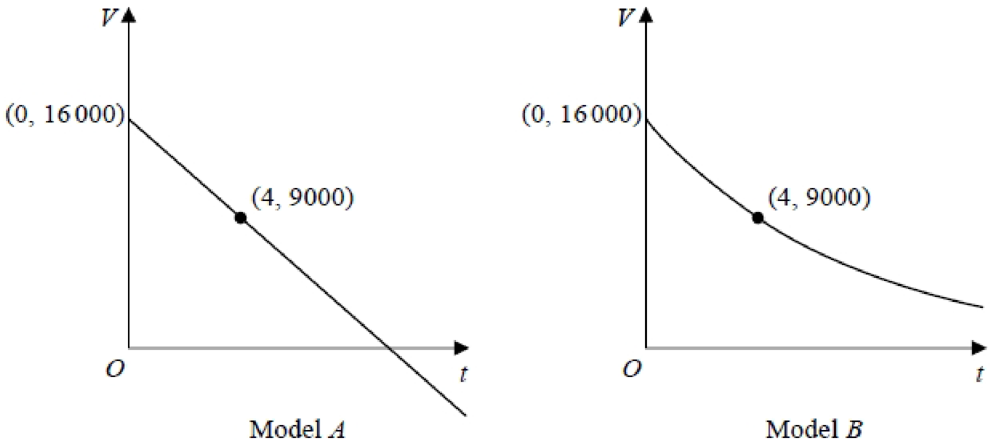
A company plans to extract oil from an oil field.

The daily volume of oil V, measured in barrels that the company will extract from this oil field depends upon the time, t years, after the start of drilling.

The company decides to use a model to estimate the daily volume of oil that will be extracted. The model includes the following assumptions:

- The initial daily volume of oil extracted from the oil field will be 16 000 barrels.
- The daily volume of oil that will be extracted exactly 4 years after the start of drilling will be 9000 barrels.
- The daily volume of oil extracted will decrease over time.

The diagram below shows the graphs of two possible models.



(a) (i) Use model A to estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(ii) Write down a limitation of using model A.

(2)

(b) (i) Using an exponential model and the information given in the question, find a possible equation for model B.

(ii) Using your answer to (b)(i) estimate the daily volume of oil that will be extracted exactly 3 years after the start of drilling.

(5)

(Total for question = 7 marks)

Q8.

In a controlled experiment, the number of microbes, N, present in a culture T days after the start of the experiment were counted.

N and T are expected to satisfy a relationship of the form

$$N = aT^b, \quad \text{where } a \text{ and } b \text{ are constants}$$

(a) Show that this relationship can be expressed in the form

$$\log_{10}N = m\log_{10}T + c$$

giving m and c in terms of the constants a and/or b.

(2)

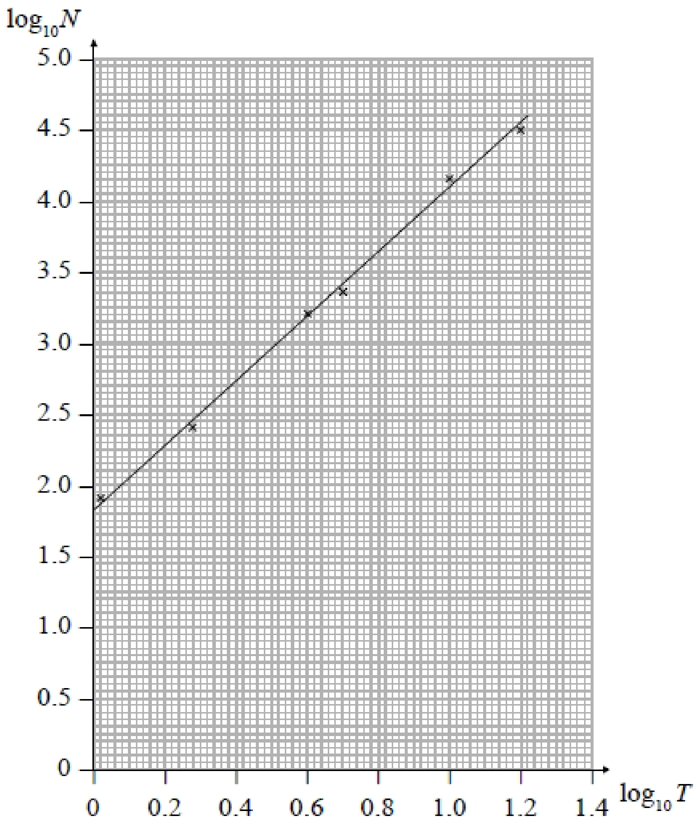


Figure 3

Figure 3 shows the line of best fit for values of $\log_{10}N$ plotted against values of $\log_{10}T$

(b) Use the information provided to estimate the number of microbes present in the culture 3 days after the start of the experiment.

(4)

(c) Explain why the information provided could not reliably be used to estimate the day when the number of microbes in the culture first exceeds 1 000 000.

(2)

(d) With reference to the model, interpret the value of the constant a.

(1)

(Total for question = 9 marks)

Q9.

The mass, m grams, of a radioactive substance, t years after first being observed, is modelled by the equation

$$m = 25e^{-0.05t}$$

According to the model,

(a) find the mass of the radioactive substance six months after it was first observed,

(2)

(b) show that $\frac{dm}{dt} = km$, where k is a constant to be found.

(2)

(Total for question = 4 marks)

Q10.

The value, £V, of a vintage car t years after it was first valued on 1st January 2001, is modelled by the equation

$$V = Ap^t \qquad \text{where } A \text{ and } p \text{ are constants}$$

Given that the value of the car was £32 000 on 1st January 2005 and £50 000 on 1st January 2012

(a) (i) find p to 4 decimal places,

(ii) show that A is approximately 24 800

(4)

(b) With reference to the model, interpret

(i) the value of the constant A,
(ii) the value of the constant p.

(2)

Using the model,

(c) find the year during which the value of the car first exceeds £100 000

(4)

(Total for question = 10 marks)

Q11.

In a simple model, the value, £V, of a car depends on its age, t, in years.

The following information is available for car A

- its value when new is £20 000
- its value after one year is £16 000

(a) Use an exponential model to form, for car A, a possible equation linking V with t.

(4)

The value of car A is monitored over a 10-year period.
Its value after 10 years is £2 000

(b) Evaluate the reliability of your model in light of this information.

(2)

The following information is available for car B

- it has the same value, when new, as car A
- its value depreciates more slowly than that of car A

(c) Explain how you would adapt the equation found in (a) so that it could be used to model the value of car B.

(1)

(Total for question = 7 marks)

Q12.

A research engineer is testing the effectiveness of the braking system of a car when it is driven in wet conditions.

The engineer measures and records the braking distance, d metres, when the brakes are applied from a speed of V km h^{-1}.

Graphs of d against V and $\log_{10} d$ against $\log_{10} V$ were plotted.

The results are shown below together with a data point from each graph.

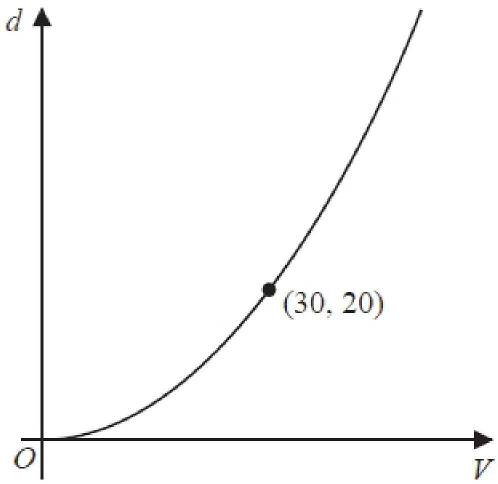


Figure 5

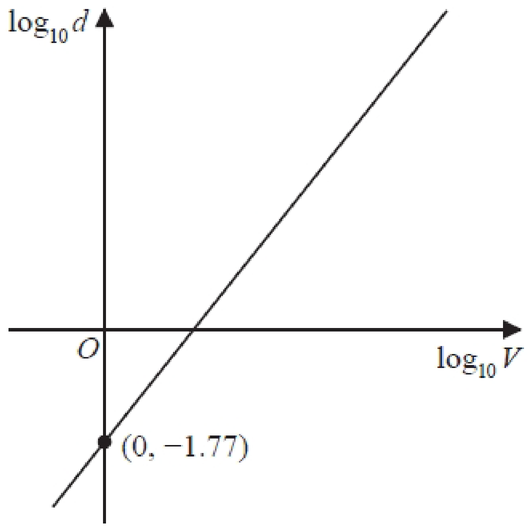


Figure 6

(a) Explain how Figure 6 would lead the engineer to believe that the braking distance should be modelled by the formula

$$d = kV^n \qquad \text{where } k \text{ and } n \text{ are constants}$$

with $k \approx 0.017$

(3)

Using the information given in Figure 5, with $k = 0.017$

(b) find a complete equation for the model giving the value of n to 3 significant figures.

(3)

Sean is driving this car at 60 km h^{-1} in wet conditions when he notices a large puddle in the road 100 m ahead. It takes him 0.8 seconds to react before applying the brakes.

(c) Use your formula to find out if Sean will be able to stop before reaching the puddle.

(3)

(Total for question = 9 marks)

Q13.

A new smartphone was released by a company.

The company monitored the total number of phones sold, n, at time t days after the phone was released.

The company observed that, during this time,

the rate of increase of n was proportional to n

Use this information to write down a suitable equation for n in terms of t.

(*You do not need to evaluate any unknown constants in your equation*.)

(Total for question = 2 marks)

Q14.

A quantity of ethanol was heated until it reached boiling point.

The temperature of the ethanol, θ °C, at time t seconds after heating began, is modelled by the equation

$$\theta = A - Be^{-0.07t}$$

where A and B are positive constants.

Given that

- the initial temperature of the ethanol was 18°C
- after 10 seconds the temperature of the ethanol was 44°C

(a) find a complete equation for the model, giving the values of A and B to 3 significant figures.

(4)

Ethanol has a boiling point of approximately 78°C

(b) Use this information to evaluate the model.

(2)

(Total for question = 6 marks)

Q15.

A scientist is studying the growth of two different populations of bacteria.

The number of bacteria, N, in the **first** population is modelled by the equation

$$N = Ae^{kt} \qquad t \geq 0$$

where A and k are positive constants and t is the time in hours from the start of the study.

Given that

- there were 1000 bacteria in this population at the start of the study
- it took exactly 5 hours from the start of the study for this population to double

(a) find a complete equation for the model.

(4)

(b) Hence find the rate of increase in the number of bacteria in this population exactly 8 hours from the start of the study.
Give your answer to 2 significant figures.

(2)

The number of bacteria, M, in the **second** population is modelled by the equation

$$M = 500e^{1.4kt} \qquad t \geq 0$$

where k has the value found in part (a) and t is the time in hours from the start of the study.

Given that T hours after the start of the study, the number of bacteria in the two different populations was the same,

(c) find the value of T.

(3)

(Total for question = 9 marks)

Mark Scheme

Q1.

Question	Scheme		Marks	AOs
(a)	$\log_{10} P = mt + c$		M1	1.1b
	$\log_{10} P = \frac{1}{200}t + 5$		A1	1.1b
			(2)	
(b)	Way 1: As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	Way 2: As $\log_{10} P = \frac{t}{200} + 5$ then $P = 10^{\left(\frac{t}{200}+5\right)} = 10^5 10^{\left(\frac{t}{200}\right)}$	M1	2.1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	$a = 10^5$ or $b = 10^{\left(\frac{1}{200}\right)}$	M1	1.1b
	so $a = 100\ 000$ **or** $b = 1.0116$		A1	1.1b
	both $a = 100\ 000$ **and** $b = 1.0116$ (awrt 1.01)		A1	1.1b
			(4)	
(c)	(i) The initial population		B1	3.4
	(ii) The proportional increase of population each year		B1	3.4
			(2)	
(d)	(i) 300000 to nearest hundred thousand		B1	3.4
	(ii) Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give $t =$		M1	3.4
	60.2 years to 3sf		A1ft	1.1b
			(3)	
(e)	Any two valid reasons- e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth		B2	3.5b
			(2)	
			(13 marks)	

Notes

(a) M1: Uses a linear equation to relate log P and t
A1: Correct use of gradient and intercept to give a correct line equation

(b) M1: Way 1: Uses logs correctly to give log equation; Way 2 Uses powers correctly to "undo" log equation and expresses as product of two powers
M1: Way 1: Identifies log b or log a or both; Way 2: identifies a or b as powers of 10
A1: Correct value for a or b
A1: Correct values for both

(c) (i) B1: Accept equivalent answers e.g. The population at $t = 0$
(ii) B1: So accept rate at which the population is increasing each year or scale factor 1.01 or increase of 1% per year

(d) (i) B1: cao
(ii) M1: as in the scheme A1ft: on their values of a and b with correct log work

(e) As given in the scheme – any two valid reasons

Q2.

Question	Scheme	Marks	AOs
(a)	For a correct equation in p or q $\quad p=10^{4.8}$ or $q=10^{0.05}$	M1	1.1b
	For $p=$ awrt 63100 or $q=$ awrt 1.122	A1	1.1b
	For correct equations in p and q $\quad p=10^{4.8}$ and $q=10^{0.05}$	dM1	3.1a
	For $p=$ awrt 63100 and $q=$ awrt 1.122	A1	1.1b
		(4)	
(b)	(i) The value of the painting on 1st January 1980	B1	3.4
	(ii) The proportional increase in value each year	B1	3.4
		(2)	
(c)	Uses $V=63100\times1.122^{30}$ or $\log V=0.05\times30+4.8$ leading to $V=$	M1	3.4
	$=$ awrt $(£)2000000$	A1	1.1b
		(2)	
			(8 marks)

Notes

(a)

M1: For a correct equation in p or q This is usually $p = 10^{4.8}$ or $q = 10^{0.05}$ but may be $\log q = 0.05$ or $\log p = 4.8$

A1: For $p =$ awrt 63100 or $q =$ awrt 1.122

M1: For linking the two equations and forming correct equations in p and q. This is usually $p = 10^{4.8}$ and $q = 10^{0.05}$ but may be $\log q = 0.05$ and $\log p = 4.8$

A1: For $p =$ awrt 63100 and $q =$ awrt 1.122 Both these values implies M1 M1

ALT I(a)

M1: Substitutes $t = 0$ and states that $\log p = 4.8$

A1: $p =$ awrt 63100

M1: Uses their found value of p and another value of t to find form an equation in q

A1: $p =$ awrt 63100 and $q =$ awrt 1.122

(b)(i)

B1: The value of the painting on 1st January 1980 (is £63 100)

Accept the original value/cost of the painting or the initial value/cost of the painting

(b)(ii)

B1: The proportional increase in value each year. Eg Accept an explanation that explains that the value of the painting will rise 12.2% a year. (Follow through on their value of q.)

Accept "the rate" by which the value is rising/price is changing. "1.122 is the decimal multiplier representing the year on year increase in value"

Do not accept "the amount" by which it is rising or "how much" it is rising by

If they are not labelled (b)(i) and (b)(ii) mark in the order given but accept any way around as long as clearly labelled " p is........... " and "q is ...."

(c)

M1: For substituting $t = 30$ into $V = pq^t$ using their values for p and q or substituting t =30 into $\log_{10} V = 0.05t + 4.8$ and proceeds to V

A1: **For awrt either** £1.99 million or £2.00 million. Condone the omission of the £ sign.

Remember to isw after a correct answer

Q3.

Question	Scheme	Marks	AOs
(a)	(£)18 000	B1	3.4
		(1)	
(b)	(i) $\frac{dV}{dt} = -3925e^{-0.25t}$	M1 A1	3.1b 1.1b
	Sets $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ * cso	A1*	3.4
	(ii) $e^{-0.25T} = 0.127... \Rightarrow -0.25T = \ln 0.127...$	M1	1.1b
	$T = 8.24$ (awrt)	A1	1.1b
	8 years 3 months	A1	3.2a
		(6)	
(c)	2 300	B1	1.1b
		(1)	
(d)	Any suitable reason such as • Other factors affect price such as condition/mileage • If the car has had an accident it will be worth less than the model predicts • The price may go up in the long term as it becomes rare • £2300 is too large a value for a car's scrap price. Most cars scrap for around £400	B1	3.5b
		(1)	
			(9 marks)

Notes
(a) **B1:** £18 000 There is no requirement to have the units **(b)(i)** **M1:** Award for making the link between gradient and rate of change. Score for attempting to differentiate V to $\frac{dV}{dt} = ke^{-0.25t}$ An attempt at both sides are required. For the left hand side you may condone attempts such as $\frac{dy}{dx}$ **A1:** Achieves $\frac{dV}{dt} = -3925e^{-0.25t}$ or $\frac{dV}{dt} = 15700\times -0.25e^{-0.25t}$ with both sides correct **A1*:** Sets $-3925e^{-0.25T} = -500$ oe and proceeds to $3925e^{-0.25T} = 500$ This is a given answer and to achieve this mark, all aspects must be seen and be correct. t must be changed to T at some point even if just at the end of their solution/proof SC: Award SC 110 candidates who simply write $-3925e^{-0.25T} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$ Or $15700\times -0.25e^{-0.25t} = -500 \Rightarrow 3925e^{-0.25T} = 500$ without any mention or reference to $\frac{dV}{dt}$ **(b)(ii)** **M1:** Proceeds from $e^{-0.25T} = A, A > 0$ using ln's to $\pm 0.25T = ..$ Alternatively takes lns first $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 - 0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ but $3925e^{-0.25T} = 500 \Rightarrow \ln 3925 \times -0.25T = \ln 500 \Rightarrow \pm 0.25T = ...$ is M0 **A1:** $T =$ awrt 8.24 or $-\frac{1}{0.25}\ln\left(\frac{20}{157}\right)$ Allow $t =$ awrt 8.24 **A1:** 8 years 3 months. Correct answer and solution only Answers obtained numerically score 0 marks. The M mark must be scored. **(c)** **B1:** 2 300 but condone £ 2 300 **(d)** **B1:** Any suitable reason. See scheme Accept "Scrappage" schemes may pay more (or less) than £ 2 300. Do not accept "does not take into account inflation" It asks for a limitation of the model so candidates cannot score marks by suggesting other suitable models " the value may fall by the same amount each year"

Q4.

Question	Scheme		Marks	AOs
(a)	Temperature = 83°C		B1	3.4
			(1)	
(b)	$18+65\mathrm{e}^{-\frac{t}{8}}=35 \Rightarrow 65\mathrm{e}^{-\frac{t}{8}}=17$		M1	1.1b
	$t=-8\ln\left(\frac{17}{65}\right)$	$\ln 65-\frac{t}{8}=\ln 17 \Rightarrow t=...$	dM1	1.1b
	$t=10.7$		A1	1.1b
			(3)	
(c)	States a suitable reason • As $t\to\infty, \theta\to 18$ from above. • The minimum temperature is 18°C		B1	2.4
			(1)	
(d)	$A+B=94$ or $A+B\mathrm{e}^{-1}=50$		M1	3.4
	$A+B=94$ and $A+B\mathrm{e}^{-1}=50$		A1	1.1b
	Full method to find at least a value for A		dM1	2.1
	Deduces $\mu=\frac{50\mathrm{e}-94}{\mathrm{e}-1}$ or accept $\mu=$ awrt 24.4		A1	2.2a
			(4)	
				(9 marks)

Notes

(a)

B1: Uses the model to state that the temperature $=83°\mathrm{C}$ Requires units as well

(b)

M1: Uses the information and proceeds to $P\mathrm{e}^{\pm\frac{t}{8}}=Q$ condoning slips

dM1: A full method using correct log laws and a knowledge that e^x and $\ln x$ are inverse functions. This cannot be scored from unsolvable equations, e.g $P>0, Q<0$. Condone one error in their solution.

A1: $t=$ awrt 10.7

(c)

B1: States a suitable reason with minimal conclusion

- As $t\to\infty, \theta\to 18$ from above.
- The minimum temperature is 18°C (so it cannot drop to 15°C)
- Substitutes $\theta=15$ (or a value between 15 and 18) into $18+65\mathrm{e}^{-\frac{t}{8}}=15$ (may be impied by $15-18=-3$ or similar) and makes a statement that $\mathrm{e}^{-\frac{t}{8}}$ cannot be less than zero or else that $\ln(-ve)$ is undefined and hence $\theta\neq 15$. All calculations must be correct
- (According to the model) the room temperature is 18°C (so cannot fall below this)

(d)

M1: Attempts to use $(0,94)$ or $(8,50)$ in order to form at least one equation in A and B
Allow this to be scored with an equation containing e^0

A1: Correct equations $A+B=94$ and $A+Be^{-1}=50$ or equivalent. $e^0=1$ must have been processed. Condone $A+B=94$ and $A+0.37B=50$ where $e^{-1}=$ awrt 0.37

dM1: Dependent upon having two equations in A and B formed from the information given. It is a full and correct method leading to a value of A. Allow this to be solved from a calculator.
Note $B=69.6..$ or $\frac{44}{1-e^{-1}} \Rightarrow A=94-"B"$

A1: Deduces that $\mu=\frac{50e-94}{e-1}$ or accept $\mu=$ awrt 24.4. Condone $y=\ldots$

Q5.

Question	Scheme		Marks	AOs
(a)	$\log_{10}V=0.072t+2.379$ $\Rightarrow V=10^{0.072t+2.379}$ $\Rightarrow V=10^{0.072t}\times10^{2.379}$	$V=ab^t$ $\Rightarrow \log_{10}V=\log_{10}a+\log_{10}b^t$ $\Rightarrow \log_{10}V=\log_{10}a+t\log_{10}b$	B1	2.1
	States either $a=10^{2.379}$ or $b=10^{0.072}$	States either $\log_{10}a=2.379$ or $\log_{10}b=0.072$	M1	1.1b
	$a=239$ or $b=1.18$	$a=239$ or $b=1.18$	A1	1.1b
	Either $V=239\times1.18^t$ or imply by $a=239, b=1.18$		A1	1.1b
			(4)	
(b)	The value of ab is the (total) number of views of the advert **1 day** after it went live.		B1	3.4
			(1)	
(c)	Substitutes $t=20$ in either equation and finds V Eg $V=239\times1.18^{20}$		M1	3.4
	Awrt 6500 or 6600		A1	1.1b
			(2)	
				(7 marks)

(a) **Condone $\log_{10}$ written $\log$ or $\lg$ written throughout the question**

B1: Scored for showing that $\log_{10} V = 0.072t + 2.379$ can be written in the form $V = ab^t$ or vice versa

Either starts with $\log_{10} V = 0.072t + 2.379$ (may be implied) and shows lines

$$V = 10^{0.072t+2.379} \text{ and } V = 10^{0.072t} \times 10^{2.379}$$

Or starts with $V = ab^t$ (implied) and shows the lines

$$\log_{10} V = \log_{10} a + \log_{10} b^t \text{ and } \log_{10} V = \log_{10} a + t\log_{10} b$$

M1: For a correct equation in a or a correct equation in b

A1: Finds either constant. Allow $a = \text{awrt } 240$ or $b = \text{awrt } 1.2$ following a correct method

A1: Correct solution: Look for $V = 239 \times 1.18^t$ or $a = 239, b = 1.18$

Note that this is NOT awrt

(b)

B1: See scheme. Condone not seeing total. Do not allow number of views at the start or similar.

(c)

M1: Substitutes $t = 20$ in either their $V = 239 \times 1.18^t$ or $\log_{10} V = 0.072t + 2.379$ and uses a correct method to find V

A1: Awrt 6500 or 6600

Q6.

Question	Scheme	Marks	AOs
(a)	35 (km^2)	B1	3.4
		(1)	
(b)	Sets their $60 = 80 - 45e^{14c} \Rightarrow 45e^{14c} = 20$	M1 A1	1.1b 1.1b
	$\Rightarrow c = \frac{1}{14}\ln\left(\frac{20}{45}\right) = \cdots - 0.0579$	dM1	3.1b
	$A = 80 - 45e^{-0.0579t}$	A1	3.3
		(4)	
(c)	Gives a suitable answer • The maximum area covered by trees is only 80km^2 • The "80" would need to be "100" • Substitutes 100 into the equation of the model and shows that the formula fails with a reason eg. you cannot take a log of a negative number	B1	3.5b
		(1)	
			(6 marks)

Notes

(a)

B1: Uses the equation of the model to find that 35 (km^2) of the reserve was covered on 1st January 2005. Do not accept eg. 35 m^2

(b)

M1: Sets their $60 = 80 - 45e^{14c} \Rightarrow Ae^{14c} = B$

A1: $45e^{14c} = 20$ or equivalent.

dM1: A full and careful method using precise algebra, correct log laws and a knowledge that e^x and $\ln x$ are inverse functions and proceeds to a value for c.

A1: Gives a complete equation for the model $A = 80 - 45e^{-0.0579t}$

(c)

B1: Gives a suitable interpretation (See scheme)

Q7.

Question	Scheme	Marks	AOs
(a)(i)	10750 barrels	B1	3.4
(ii)	Gives a valid limitation, for example • The model shows that the daily volume of oil extracted would become negative as t increases, which is impossible • States when $t = 10, V = -1500$ which is impossible • States that the model will only work for $0 \leqslant t \leqslant \frac{64}{7}$	B1	3.5b
		(2)	
(b)(i)	Suggests a suitable exponential model, for example $V = Ae^{kt}$	M1	3.3
	Uses $(0,16000)$ and $(4,9000)$ in $\Rightarrow 9000 = 16000e^{4k}$	dM1	3.1b
	$\Rightarrow k = \frac{1}{4}\ln\left(\frac{9}{16}\right)$ awrt -0.144	M1	1.1b
	$V = 16000e^{\frac{1}{4}\ln\left(\frac{9}{16}\right)t}$ or $V = 16000e^{-0.144t}$	A1	1.1b
(ii)	Uses their exponential model with $t = 3 \Rightarrow V =$ awrt 10 400 barrels	B1ft	3.4
		(5)	
			(7 marks)

Notes:	
(a)(i)	
B1:	10750 barrels
(a)(ii)	
B1:	See scheme
(b)(i)	
M1:	Suggests a suitable exponential model, for example $V = Ae^{kt}$, $V = Ar^t$ or any other suitable function such as $V = Ae^{kt} + b$ where the candidate chooses a value for b.
dM1:	Uses both $(0,16000)$ and $(4,9000)$ in their model.
	With $V = Ae^{kt}$ candidates need to proceed to $9000 = 16000e^{4k}$
	With $V = Ar^t$ candidates need to proceed to $9000 = 16000r^4$
	With $V = Ae^{kt} + b$ candidates need to proceed to $9000 = (16000 - b)e^{4k} + b$ where b is given as a positive constant and $A + b = 16000$.
M1:	Uses a correct method to find all constants in the model.
A1:	Gives a suitable equation for the model passing through (or approximately through in the case of decimal equivalents) both values $(0,16000)$ and $(4,9000)$. Possible equations for the model could be for example
	$V = 16000e^{-0.144t}$ $\quad$ $V = 16000 \times (0.866)^t$ $\quad$ $V = 15800e^{-0.146t} + 200$
(b)(ii)	
B1ft:	Follow through on their exponential model

Q8.

Question	Scheme	Marks	AOs
(a)	$N = aT^b \Rightarrow \log_{10} N = \log_{10} a + \log_{10} T^b$	M1	2.1
	$\Rightarrow \log_{10} N = \log_{10} a + b\log_{10} T$ so $m = b$ **and** $c = \log_{10} a$	A1	1.1b
		(2)	
(b)	Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** $b = \text{gradient}$	M1	3.1b
	Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** $b = \text{gradient}$	M1	1.1b
	Uses $T = 3$ in $N = aT^b$ with their a and b	M1	3.1b
	Number of microbes ≈ 800	A1	1.1b
		(4)	
(c)	$N = 1000000 \Rightarrow \log_{10} N = 6$	M1	3.4
	We cannot 'extrapolate' the graph and assume that the model still holds	A1	3.5b
		(2)	
(d)	States that 'a' is the number of microbes 1 day after the start of the experiment	B1	3.2a
		(1)	
			(9 marks)

Notes:

(a)

M1: Takes logs of both sides and shows the addition law

M1: Uses the power law, writes $\log_{10} N = \log_{10} a + b\log_{10} T$ and states $m = b$ **and** $c = \log_{10} a$

(b)

M1: Uses the graph to find either a or b $a = 10^{\text{intercept}}$ **or** $b = \text{gradient}$. This would be implied by the sight of $b = 2.3$ **or** $a = 10^{1.8} \approx 63$

M1: Uses the graph to find both a and b $a = 10^{\text{intercept}}$ **and** $b = \text{gradient}$. This would be implied by the sight of $b = 2.3$ **and** $a = 10^{1.8} \approx 63$

M1: Uses $T = 3 \Rightarrow N = aT^b$ with their a and b. This is implied by an attempt at $63 \times 3^{2.3}$

A1: Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work.

There is an alternative to this using a graphical approach.

M1: Finds the value of $\log_{10} T$ from T=3. Accept as $T = 3 \Rightarrow \log_{10} T \approx 0.48$

M1: Then using the line of best fit finds the value of $\log_{10} N$ from their "0.48"

Accept $\log_{10} N \approx 2.9$

M1: Finds the value of N from their value of $\log_{10} N$ $\log_{10} N \approx 2.9 \Rightarrow N = 10^{'2.9'}$

A1: Accept a number of microbes that are approximately 800. Allow 800 ± 150 following correct work

(c)
M1 For using $N = 1000000$ and stating that $\log_{10} N = 6$
A1: Statement to the effect that "we only have information for values of $\log N$ between 1.8 and 4.5 so we cannot be certain that the relationship still holds". "We cannot extrapolate with any certainty, we could only interpolate" There is an alternative approach that uses the formula.
M1: Use $N = 1000000$ in their $N = 63 \times T^{2.3} \Rightarrow \log_{10} T = \dfrac{\log_{10}\left(\frac{1000000}{63}\right)}{2.3} \approx 1.83$.
A1: The reason would be similar to the main scheme as we only have $\log_{10} T$ values from 0 to 1.2. We cannot 'extrapolate' the graph and assume that the model still holds
(d)
B1: Allow a numerical explanation $T = 1 \Rightarrow N = a1^b \Rightarrow N = a$ giving a is the value of N at $T = 1$

Q9.

Question	Scheme	Marks	AOs
(a)	Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times 0.5}$	M1	3.4
	$\Rightarrow m = 24.4\text{g}$	A1	1.1b
		(2)	
(b)	States or uses $\dfrac{d}{dt}\left(e^{-0.05t}\right) = \pm C e^{-0.05t}$	M1	2.1
	$\dfrac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$	A1	1.1b
		(2)	
			(4 marks)

Notes:
(a)
M1: Substitutes $t = 0.5$ into $m = 25e^{-0.05t} \Rightarrow m = 25e^{-0.05\times 0.5}$
A1: $m = 24.4\text{g}$ An answer of $m = 24.4\text{g}$ with no working would score both marks
(b)
M1: Applies the rule $\dfrac{d}{dt}\left(e^{kx}\right) = k e^{kx}$ in this context by stating or using $\dfrac{d}{dt}\left(e^{-0.05t}\right) = \pm C e^{-0.05t}$
A1: $\dfrac{dm}{dt} = -0.05 \times 25e^{-0.05t} = -0.05m \Rightarrow k = -0.05$

Q10.

Question	Scheme	Marks	AOs
(a)	(i) Method to find p Eg. Divides $32000 = Ap^4$ by $50000 = Ap^{11}$ $p^7 = \frac{50000}{32000} \Rightarrow p = \sqrt[7]{\frac{50000}{32000}} = ...$	M1	3.1a
	$p = 1.0658$	A1	1.1b
	(ii) Substitutes their $p = 1.0658$ into either equation and finds A $A = \frac{32000}{'1.0658'^4}$ or $A = \frac{50000}{'1.0658'^{11}}$	M1	1.1b
	$A = 24795 \to 24805 \simeq 24\,800$ *	A1*	1.1b
		(4)	
(b)	A / $(£)24\,800$ is the value of the car on 1st January 2001	B1	3.4
	p /1.0658 is the factor by which the value rises each year. Accept that the value rises by 6.6 % a year (ft on their p)	B1	3.4
		(2)	
(c)	Attempts $100000 = '24800' \times '1.0658'^t$		
	$'1.0658'^t = \frac{100000}{24800}$	M1	3.4
	$t = \log_{1.0658}\left(\frac{100000}{24800}\right)$	dM1	1.1b
	$t = 21.8$ or 21.9	A1	1.1b
	cso 2022	A1	3.2a
		(4)	
		(10 marks)	

(a) (i)

M1: Attempts to use both pieces of information within $V = Ap^t$, eliminates A correctly and solves an equation of the form $p^n = k$ to reach a value for p.

Allow for slips on the 32 000 and 50 000 and the values of t.

A1: $p =$ awrt 1.0658

Both marks can be awarded from incorrect but consistent interpretations of t. Eg. $32000 = Ap^5, 50000 = Ap^{12}$

(a)(ii)

M1: Substitutes their $p = 1.0658$ into either of their equations and finds A

Eg $A = \frac{32000}{1.0658^4}$ or $A = \frac{50000}{1.0658^7}$ but you may follow through on incorrect equations from part (i)

A1*: Shows that A is between 24 795 and 24 805 before you see ' =24 800' or ' ≈ 24800 '. Accept with or without units.

An alternative to (ii) is to start with the given answer.

M1: Attempts $24800 \times '1.0658'^4 = (32000.34)$

A1: 24800×1.0658^4, achieves a value between 31095 and 32005 followed by $\approx 32\,000$ hence A must be $\approx 24\,800$

(b)

B1: States that A is the value of the car on 1st January 2001.

The statement must reference **the car**, its **cost/value**, and **"0" time**

Allow 'it is the initial value of the car" "it is the cost of the car at $t = 0$" "it is the cars starting value"

B1: States that p is the rate at which the value of the car rises each year.

The statement must reference **a yearly rate** and **an increase in value or multiplier**.

They could reference the 1.0658 Eg "The cars value rises by 6.5 % each year."

Allow "p is the rate the cars value is rising each year" "it is the proportional increase in value of the car each year" "the factor by which the value of the car is rising each year" 'its value appreciates by 6.5% per year' Allow ' the value of the car multiplies by p each year'

Do not allow "by how much the value of the car rises each year " or "it is the rate of inflation"

(c)

M1: Uses the model $100000 = '24800' \times '1.0658'^{t}$ and proceeds to their $'1.0658'^{t} = k$

Allow use of any inequality here.

dM1: For the complete method of (i) using the information given with their equation of the model **and** (ii) translating the situation into a correct method to find 't'

A1: $(t) =$ awrt 21.8 or 21.9 or $\log_{1.0658}\left(\dfrac{100000}{24800}\right)$ oe

A1: States in the year 2022. A candidate using a GP formula can be awarded full marks

Allow different methods in part (c).

Eg Via GP a formula

M1: $24800 \times '1.0658'^{n-1} = 100000 \Rightarrow '1.0658'^{n-1} = K$

dM1: Uses a correct method to find n.

A2: 2022

Via (trial and improvement)

M1: Uses the model by substituting integer values of t into their $V = Ap^t$ so that for $t = n, V < 100\,000$ or $t = n+1, V > 100\,000$

(So for the correct A and p this would be scored for $t = 21, V \approx £95000$ or $t = 21, V \approx £101\,000$

dM1: For a complete method showing that this is the least value. So both of the above values

A1: Allow for 22 following correct and accurate results (awrt nearest £1000 is sufficient accuracy)

A1: As before

Q11.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	$V = Ae^{-kt}$	M1	This mark is given for suggesting a suitable exponential model for V in terms of t
	When $t = 0$ and $V = 20\,000$, $A = 20\,000$	M1	This mark is given for using the model to show the initial value for A is £20 000
	When $t = 1$ and $V = 16\,000$, $16\,000 = 20\,000e^{-1k}$ $k = \ln 0.8 = -0.223$	M1	This mark is given for using the value of the car after one year to find a value for k
	$V = 20\,000e^{-0.223t}$	A1	This mark is given for finding a fully correct exponential model
(b)	When $t = 10$, $V = £2150$	M1	This mark is given for finding a value for V when $t = 10$
	This model is reliable since the value £2150 is close to £2000	A1	This mark is given for a valid statement comparing the two possible values of the car after 10 years
(c)	For example: The value of k should be increased (e.g. $V = 20\,000e^{-0.1t}$) A constant should be added (e.g. $V = 20\,000e^{-0.223t} + 2000$)	B1	This mark is given for a statement suggesting a valid adaptation

Q12.

Part	Working or answer an examiner might expect to see	Mark	Notes
(a)	If $d = kV^n$, then $\log_{10} d = \log_{10} k + n \log_{10} V$	M1	This mark is given for
	Plotting $\log_{10} d$ against $\log_{10} V$ will result in a straight line with gradient n and intercept $\log_{10} k$	A1	This mark is given for an explanation of why the second graph shows that $d = kV^n$
	$\log_{10} k = -1.77$ $k = 10^{-1.77} = 0.01698\ldots \approx 0.017$	A1	This mark is for showing fully that $k \approx 0.017$
(b)	$d = kV^n$ When $V = 30$, $d = 20$ and $k = 0.17$ then $20 = 0.017 \times 30^n$	M1	This mark is given for substituting in the formula as a method to find the value of n
	$n \log 30 = \log \left(\dfrac{20}{0.017}\right)$	M1	This mark is given for a correct expression for n
	$n = 2.08$ to 3 significant figures $d = 0.017 \times V^{2.08}$	A1	This mark is given for finding a correct value of n to 3 significant figures and writing a complete equation for the model
(c)	$\dfrac{60}{3600} \times 0.8 \times 1000 = 13.33$ m	M1	This mark is given for a method to find the distance, in metres, covered in the reaction time of 0.8 seconds
	$d = 0.017 \times 60^{2.08} = 84.92$ m	M1	This mark is given for a method to use the formula to find the stopping distance
	13.33 m + 84.92 m = 98.25 m Sean will be able to stop before reaching the puddle	A1	This mark is given for finding a correct value of the total stopping distance and giving a valid conclusion
			(Total 9 marks)

Q13.

Question	Scheme	Marks	AOs
	Any equation involving an exponential of the correct form. See notes	M1	3.1b
	$n = Ae^{kt}$ (where A and k are positive constants)	A1	1.1b
		(2)	
			(2 marks)
Notes:			

M1: Any equation of the correct form, involving n and an exponential in t.

So allow for example $n = e^{\pm t}$, $n = Ae^{\pm t}$, $n = Ae^{\pm kt}$ condoning $n = A + Be^{\pm t}$

Condone an intermediate form where n has not been made the subject.

E.g Allow $\ln n = kt + c$ but also condone $\ln n = kt$

A1: E.g. $n = Ae^{kt}$, $n = e^{kt+c}$, $n = e^{kt}e^{c}$ There is no requirement to state that A and k are positive constants

Note that the two constants need to be different.

Mark the final answer so $n = e^{kt+c}$ followed by $n = e^{kt} + e^{c}$ o.e. $n = e^{kt} + A$ such as is M1 A0

You may see solutions that don't include "e".

These are fine so you can include versions of $n = Ak^t$ using the same marking criteria as above

FYI $\frac{dn}{dt} = Ak^t \times \ln k = \ln k \times n$ so $\frac{dn}{dt} \propto n$

Q14.

Question	Scheme	Marks	AOs
(a)	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ or $t = 10,\ \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$	M1	3.1b
	$t = 0,\ \theta = 18 \Rightarrow 18 = A - B$ and $t = 10,\ \theta = 44 \Rightarrow 44 = A - Be^{-0.7}$ and $\Rightarrow A = ..., B = ...$	M1	3.1a
	At least one of: $A = 69.6$, $B = 51.6$ but allow awrt 70/awrt 52	A1 **M1 on EPEN**	1.1b
	$\theta = 69.6 - 51.6e^{-0.07t}$	A1	3.3
		(4)	
(b)	The maximum temperature is "69.6"(°C) (according to the model) (The model has an) upper limit of "69.6"(°C) (The model suggests that) the boiling point is "69.6"(°C)	B1ft	3.4
	Model is not appropriate as 69.6(°C) is much lower than 78(°C)	B1ft	3.5a
		(2)	
			(6 marks)

Notes:

(a)

M1: Makes the first key step in the solution of the problem. Substitutes $t = 0$ and $\theta = 18$ **or** $t = 10$ and $\theta = 44$ into the equation of the model to obtain an equation connecting A and B.
Note that $18 = A - Be^0$ scores M0 unless $18 = A - B$ is seen or implied later.
If they do not obtain an equation in A and B using the first conditions e.g. they have $18 = A - 1$ then they can score this mark if they substitute $A = 19$ directly into $44 = A - Be^{-0.7}$ as an equation in A and B is implied.

M1: Substitutes $t = 0$ and $\theta = 18$ **and** $t = 10$ and $\theta = 44$ to obtain 2 equations connecting A and B **and** then proceeds to solves their equations in A and B simultaneously to obtain values for both constants. Do not be too concerned with the processing as long as values for A and B are obtained.

A1(M1 on EPEN): For $A =$ awrt 70 **or** $B =$ awrt 52

A1: For $\theta = 69.6 - 51.6e^{-0.07t}$ **Must be a <u>fully correct equation as shown</u> but allow recovery if seen in (b).**
Note that some candidates evaluate e^0 as 0 and so obtain $A = 18$ and then write $44 = 18 - Be^{-0.7}$ and solve for B. Such attempts can score M1M0A0A0 only.

(b)

B1ft: Identifies A as the boiling point/maximum temperature in the model. Follow through their A.

B1ft: Makes a valid conclusion (valid/not valid, good/not good etc.) that refers to the 78 and includes a reference to a significant/large difference

Alternative provided their $A < 78$

B1ft: $\theta = 69.6 - 51.6e^{-0.07t} = 78 \Rightarrow 51.6e^{-0.07t} = 69.6 - 78 = -8.4$

$\Rightarrow e^{-0.07t} = -\dfrac{7}{43}$ and $\ln\left(-\dfrac{7}{43}\right)$ and makes a reference to the fact that the equation cannot be solved or e.g. cannot take log of a negative number. You can condone numerical slips in the calculation.

B1ft: Model is not appropriate as 69.6(°C) is much lower than 78(°C)

Minimum for both marks: The model is not appropriate as "69.6"(°C) is much lower than 78(°C)

Note that these marks are not available if their equation is solvable. Note also that B0B1 is not possible.

Q15.

Question	Scheme	Marks	AOs
(a)	$A = 1000$	B1	3.4
	$2000 = 1000\mathrm{e}^{5k}$ or $\mathrm{e}^{5k} = 2$	M1	1.1b
	$\mathrm{e}^{5k} = 2 \Rightarrow 5k = \ln 2 \Rightarrow k = \ldots$	M1	2.1
	$N = 1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $N = 1000\mathrm{e}^{0.139t}$	A1	3.3
		(4)	
(b)	$\frac{\mathrm{d}N}{\mathrm{d}t} = 1000 \times \left(\frac{1}{5}\ln 2\right)\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)t}$ or $\frac{\mathrm{d}N}{\mathrm{d}t} = 1000 \times 0.139\mathrm{e}^{0.139t}$ $\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8} = 1000 \times \left(\frac{1}{5}\ln 2\right)\mathrm{e}^{8 \times \frac{1}{5}\ln 2}$ or $\left(\frac{\mathrm{d}N}{\mathrm{d}t}\right)_{t=8} = 1000 \times 0.139\mathrm{e}^{0.139 \times 8}$	M1	3.1b
	= awrt 420	A1	1.1b
		(2)	
(c)	$500\mathrm{e}^{1.4 \times \left(\frac{1}{5}\ln 2\right)T} = 1000\mathrm{e}^{\left(\frac{1}{5}\ln 2\right)T}$ or $500\mathrm{e}^{1.4 \times "0.139"t} = 1000\mathrm{e}^{"0.139"t}$	M1	3.4
	Correct method of getting a linear equation in T E.g. $0.08T\ln 2 = \ln 2$ or $1.4 \times "0.339"T = \ln 2 + "0.339"t$	M1	2.1
	$T = 12.5$ hours	A1	1.1b
		(3)	
		(9 marks)	
	Notes		

Mark as one complete question. Marks in (a) can be awarded from (b)

(a)

B1: Correct value of A for the model. Award if equation for model is of the form $N = 1000\mathrm{e}^{\dots t}$

M1: Uses the model to set up a correct equation in k. Award for substituting $N = 2000, t = 5$ following through on their value for A.

M1: Uses correct ln work to solve an equation of the form $a\mathrm{e}^{5k} = b$ and obtain a value for k

A1: Correct equation of model. Condone an ambiguous $N = 1000\mathrm{e}^{\frac{1}{5}\ln 2t}$ unless followed by something incorrect. Watch for $N = 1000 \times 2^{\frac{1}{5}t}$ which is also correct

(b)

M1: Differentiates $\alpha\mathrm{e}^{kt}$ to $\beta\mathrm{e}^{kt}$ and substitutes $t = 8$ (Condone $\alpha = \beta$ so long as you can see an attempt to differentiate)

A1: For awrt 420 (2sf).

(c)

M1: Uses both models to set up an equation in T using their value for k, but also allow in terms of k

M1: Uses correct processing using lns to obtain a linear equation in T (or t)

A1: Awrt 12.5

Answers to (b) and (c) appearing without working (i.e. from a calculator).

It is important that candidates show sufficient working to make their methods clear.

(b) If candidate has for example $N = 1000\mathrm{e}^{0.139t}$, and then writes at $t = 8$ $\frac{\mathrm{d}N}{\mathrm{d}t} =$ awrt 420 award both marks. Just the answer from a correct model equation score SC 1,0.

(c) The first M1 should be seen E.g $500\mathrm{e}^{1.4\times\text{"}0.139\text{"}t} = 1000\mathrm{e}^{\text{"}0.139\text{"}t}$

If the answer $T = 12.5$ appears without any further working score SC M1 M1 A0
